Analytic noise model for efficient approximation of Itô integrals

Andrzej Kałuża, Paweł M. Morkisz, Paweł Przybyłowicz

In the talk we present results on approximation of stochastic integrals of the following form

$$\mathcal{I}(X, W) = \int_0^T X(t)dW(t),$$

where $T > 0$, $W = \{W(t)\}_{t\geq 0}$ is a standard one-dimensional Wiener process, and $X = \{X(t)\}_{t\in[0,T]}$ belongs to a class of progressively measurable stochastic processes that are Hölder continuous in the rth mean.

Inspired by increasingly popularity of computations with low precision (used on Graphics Processing Units – GPUs and standard Computer Processing Units – CPU for significant speedup), we introduce suitable analytic noise model of standard noisy information about X and W. In this model we show that the upper bounds on the error of the Riemann-Maruyama quadrature are proportional to $n^{-\varrho} + \delta_1 + \delta_2$, where n is a number of noisy evaluations of X and W, $\varrho \in (0, 1]$ is a Hölder exponent of X, and $\delta_1, \delta_2 \geq 0$ are precision parameters for values of X and W, respectively. Moreover, we show that the error of any algorithm based on at most n noisy evaluations of X and W is at least $C(n^{-\varrho} + \delta_1)$. We also report numerical experiments performed on both CPU and GPU that confirm our theoretical findings. We also present some computational performance comparison between those two architectures.