Tractability of multivariate problems or how to fight with the curse of dimensionality

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We consider multivariate problems defined on spaces of d variate functions. We assume that we can use arbitrary linear functionals or only function values. The information complexity $n(\varepsilon, d)$ is defined as the minimal number of linear functionals (or function values) needed to compute an ε approximation.

For many problems the information complexity $n(\varepsilon, d)$ is an exponential function of d, which is called the curse of dimensionality. This usually holds in the worst case setting and may hold independently of smoothness of functions.

Tractability means that the information complexity is not an exponential function of ε^{-1} and d. Various notions of tractability are defined by specifying what kind of $n(\varepsilon, d)$ is expected. For instance, polynomial tractability means that $n(\varepsilon, d)$ can be bounded by a polynomial in ε^{-1} and d.

The curse of dimensionality of the original problem can be broken by

1. switching to weighted spaces with appropriety decreasing weights, or
2. switching to spaces with increasing smoothness with respect to successive variables, or
3. switching to a more lenient setting than the worst case setting.