I. INTRODUCTION

Modal logic was first introduced by philosophers to study the deductive behaviour of the expressions it is necessary that and it is possible that. Nowadays, it is widely used in several areas of computer science, including formal verification and artificial intelligence.

Syntactically, modal logic extends propositional logic by two unary operators: ♦ and □. The formal semantics is usually given in terms of Kripke structures. Basically, a Kripke structure is a directed graph, called a frame, together with a valuation of propositional variables. Vertices of this graph are called worlds. For each world truth values of all propositional variables are defined independently. In this semantics, ♦ϕ means the current world is connected to some world in which ϕ is true; and □ϕ, equivalent to ¬♦¬ϕ, means ϕ is true in all worlds to which the current world is connected.

Basic modal logic, as defined above, is rather weak and has limited applications. Thus, numerous variations and extensions of this formalism, including temporal and description logics, have been proposed and investigated. They vary in the complexity of the satisfiability problem, which usually lies between NP and 2EXPSPACE. However, even some simple extensions may lead to undecidability (see e.g. [1]).

A popular approach to extend modal logic is to restrict the class of admissible Kripke frames. For example, we may require them to be reflexive and transitive (which corresponds to modal logic S4) or enforce their transition relation to be an equivalence relation (S5). There are several ways in which such restrictions can be imposed.

We focus on restricting the class of the admissible frames by a first-order logic sentence that uses a single binary relation R, which is interpreted as the transition relation. Eg., the sentence ∀xyz.xRy ∧ yRz ⇒ xRz defines the class of all transitive frames. Modal logic over a class of frames definable by a first-order logic sentence is called an elementary modal logic.

The main goal of our research is to classify all elementary modal logics with respect to decidability and complexity of their satisfiability problems.

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REFERENCES