MR856711 (88a:33023) 33A65 (33A45)
Lewanowicz, S. (PL-WROC)
Properties of the polynomials associated with the Jacobi polynomials.
Math. Comp. **47** (1986), *no. 176,* 669–682.

If $P_n(x)$ are monic orthogonal polynomials associated with the weight function $w(x)$ on $[a,b]$ then the associated polynomials $q_n(x)$ are defined by $q_n(x)=\int_a^b((p_n(x)-p_n(t))/(x-t))w(t)\,dt$. The author proves identities satisfied by the polynomials $W_n(x,\alpha,\beta)$ associated with the Jacobi polynomials $P_n(x,\alpha,\beta)$. For example, he proves that the W_n satisfy the nonhomogeneous second-order differential equation

$$(1-x^2)y''+[(\alpha+\beta-2)x+\alpha-\beta]y'+(n+1)(n+\alpha+\beta)y= 2(\alpha+\beta+1)\frac{d}{dx}P_n(x,\alpha,\beta).$$

He also proves various differential-difference identities for the W_n as well as giving explicit representations of the W_n of the form $W_n(x,\alpha,\beta)=\sum_{k=0}^{n-1}a_{nk}((1-x)/2)^k$, $W_n(x,\alpha,\beta)=\sum_{k=0}^{n-1}b_{nk}P_k(x,\alpha,\beta)$. In addition, he proves that the beta-type integral $J_n(\alpha,\beta)=\int_{-1}^1(1-t)^\mu(1+t)^\nu W_n(t,\alpha,\beta)\,dt$ satisfies a second-order linear difference equation. In the case of the associated Gegenbauer (ultraspherical) polynomials, these identities simplify, and he gives these simpler formulas.

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