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FORMULAE RELATING LITTLE q-JACOBI, q-HAHN AND q-BERNSTEIN POLYNOMIALS: APPLICATION TO THE q-BÉZIER CURVE EVALUATION

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Abstract. Coefficients in the expansion of the q–Bernstein polynomials of degree n,

$$b_i^n(x;q) := \begin{bmatrix} n \\ i \end{bmatrix}_q x^i\,(x;\,q)_{n-i}\,, \quad 0 \le i \le n,$$

in terms of little q–Jacobi polynomials,

$$p_k(x;\alpha,\beta|q) = (-1)^k q^{\binom{k}{2}} \frac{(\alpha q;q)_k}{(\alpha\beta q^{k+1};q)_k}\, {}_2\phi_1\left(\begin{matrix} q^{-k},\ \alpha\beta q^{k+1} \\ \alpha q \end{matrix}\ \Big|\, q;qx\right), \qquad k \ge 0,$$

are given explicitly as evaluations of the q–Hahn polynomials,

$$Q_k(q^{-x};a,b,N|q) := {}_3\phi_2\left(\begin{matrix} q^{-k}, abq^{k+1}, q^{-x} \\ aq, q^{-N} \end{matrix}\ \Big|\, q;q\right), \quad k = 0,1,\dots,N, \quad N \in \mathbb{N}.$$

Namely, we show that

$$\begin{aligned} b_i^n(x;q) \ &= \ (-1)^i(-\beta q)^n \begin{bmatrix} n \\ i \end{bmatrix}_q q^{\binom{n-i}{2}} \\ &\times \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{(\alpha q^{k+1};\,q)_{n-k}}{(\alpha\beta q^{2k+2};\,q)_{n-k}} Q_{n-k}\left(q^{i-n};\frac{1}{\alpha q^{n+1}},\frac{1}{\beta q^{n+1}},n\Big|q\right) p_k\left(\frac{x}{\beta q};\alpha,\beta\Big|\,q\right). \end{aligned}$$

Also, we show that the well-known Ciesielski's formula, relating Bernstein, Jacobi and Hahn polynomials, is a limit form of the above equation.

An *inverse formula*, involving dual q–Hahn polynomials, is also given.

These results are used to derive a procedure for obtaining M points on a q–Bézier curve of degree n in $\mathbb{R}^d$, with a cost of $O(n^2 + Mnd)$ operations.

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