Representations for the first associated q-classical orthogonal polynomials

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Abstract

Let $\{p_k(x;q)\}$ be any system of the q-classical orthogonal polynomials, and let ϱ be the corresponding weight function, satisfying the q-difference equation $\mathbf{D}_q(\sigma\varrho) = \tau\varrho$, where σ and τ are polynomials of degree at most 2 and exactly 1, respectively. Further, let $\{p_k^{(1)}(x;q)\}$ be associated polynomials of the polynomials $\{p_k(x;q)\}$. Explicit forms of the coefficients $b_{n,k}$ and $c_{n,k}$ in the expansions

$$p_{n-1}^{(1)}(x;q) = \sum_{k=0}^{n-1} b_{n,k}\vartheta_k(x),$$
$$p_{n-1}^{(1)}(x;q) = \sum_{k=0}^{n-1} c_{n,k}p_k(x;q)$$

are given in terms of basic hypergeometric functions. Here $\vartheta_k(x)$ equals x^k if $\sigma^+(0) = 0$, or $(x;q)_k$ if $\sigma^+(1) = 0$, where $\sigma^+(x) := \sigma(x) + (q-1)x\tau(x)$. The most important representatives of those two classes are the families of little q-Jacobi and big q-Jacobi polynomials, respectively.

Writing the second-order nonhomogeneous q-difference equation satisfied by $p_{n-1}^{(1)}(x;q)$ in a special form, recurrence relations (in k) for $b_{n,k}$ and $c_{n,k}$ are obtained in terms of σ and τ.