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Report on:
"Approximation Algorithms for Planar Node-Weighted Network Design and Facility Location" – PhD Thesis of Mateusz Lewandowski
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The PhD thesis of Mateusz Lewandowski focuses on developing *approximation algorithms* for some fundamental *network design* problems.

*A*pproximation algorithms are efficient algorithms that deliver provably near-optimal solutions. Approximation algorithms are an important and active area of research in Theoretical Computer Science (TCS), whose popularity constantly increased in the past 60 years. The present work focuses on developing such algorithms for network design problems such as (variants of) the *Steiner Tree* problem, and the *Facility Location* problem. These are among the most important and fundamental problems in combinatorial optimization.

Roughly speaking, both problems are defined on a given network, represented as a weighted graph. The *Steiner Tree* problem deals with connecting a subset of terminals of this network at minimum cost. In the *Facility Location* problem, we want install some facilities on a subset of the nodes and connect them to clients, while minimizing the total installation and connection cost. Both Steiner Tree and Facility Location have a tremendous set of applications, and as such, they have been widely studied in the literature from both a theoretical and a practical perspective. In particular, the approximability of these problems is a pivotal research topic.

The present thesis fits in this context, and provides the following main contributions.

(1) Description of a result from the author's Master thesis: Prize-collecting node-weighted Steiner tree problem on planar graphs.

The thesis starts with a chapter describing a result on a variant of Steiner Tree, called the Node-weighted Prize-collecting Steiner tree problem (NWPCST) on planar graphs. In NWPCST, we are given a graph $G = (V, E)$, where each node $v \in V$ has a non-negative weight and a non-negative penalty. The goal is to compute a subtree T of G minimizing the sum of the weights of the nodes $v \in T$ and the sum of the penalties of the nodes $v \notin T$. NWPCST is NP-hard, and even hard-to-approximate within a factor better than $O(\log n)$. However, constant approximation factors are possible on *planar* graphs: these are graphs that can be drawn on a plane without edge crossings. Prior to the author's work, the best approximation factor for NWPCST on planar graphs was 2.93 [Moldenhauer'14].
The main results of this chapter are as follows. First, the chapter describes a 3-approximation algorithm for the above problem, that has the so-called *Lagrangian multiplier preserving* (LMP) property. The latter is a property of crucial importance in the design of approximation algorithms which rely on Lagrangian relaxation and the primal-dual method, e.g. for quota-variants of node-weighted Steiner Tree. Second, it presents a Linear Programming (LP) based 2.88-approximation algorithm, which improves over the previously best known factor. Third, it describes a 4-approximation algorithm based on the primal-dual scheme,

which holds for the more general *forest* version.
As already mentioned, it is my understanding that the results in this chapter are based on the author's Master's thesis.

(2) Node-weighted k-MST on planar graphs.

The node-weighted k-MST problem asks to find a minimum weight connected subgraph of a given network, which contains at least k nodes. The main result of this chapter is showing a $(4/3 \cdot r + \varepsilon)$-approximation algorithm for this problem, in any class of graphs where the LMP-approximation algorithm for NWPCST described in the previous paragraph yields an approximation factor of r. For planar graphs, this translates into a $(4 + \varepsilon)$-approximation. The algorithm relies on a Lagrangian relaxation approach (see e.g. [Chudack et al.'04]). One novelty introduced by the author here is a clever "merging procedure" (which, in the context of Lagrangian relaxations, refers to the task of augmenting some solution by picking additional vertices from a larger solution). Interestingly, the author draws a nice connection between this problem and partial cover problems as described in [Könemann at al. '11]. This connection allows the author to show that the approximation bound given in this thesis is essentially best possible for techniques that rely on using an LMP algorithm for NWPCST as a black box.

(3) Edge-weighted Steiner Tree on map graphs.

The edge-weighted Steiner tree is the "standard" Steiner Tree variant, mentioned at the beginning of this report: given a graph $G = (V, E)$ with non-negative edge weights and a set of terminal nodes, find a minimum-weight tree T spanning the terminals. On planar graphs, Steiner Tree admits a *Polynomial time Approximation Scheme* (PTAS). Namely, one can compute in polynomial time a $(1 + \varepsilon)$-approximate solution for any fixed $\varepsilon > 0$. In this chapter, the author provides a PTAS for the edge-weighted Steiner Tree problem on uniform map graphs, which generalize planar graphs. These graphs are defined as the intersection graph of regions in the plane, where two regions are adjacent if they share at least one point. There is an interesting characterization of map graphs, using the concept of half-square operation on bipartite graphs. Via this notion, the author draws a connection between instances of the edge-weighted Steiner Tree on map graphs and some instances of the node-weighted Steiner Tree problem on planar graphs. Indeed, the author provides a PTAS for this subset of instances of planar node-weighted Steiner Tree. The algorithm builds upon the classical PTAS scheme of [Borradaile et al.'09], but in order to generalize the scheme to this setting the author needs to overcome several challenges.

(4) Facility Location with Penalties.

This chapter focuses on a generalization of the classical Uncapacitated Facility Location (UFL). As in the classical setting, we are given a set of clients that wish to be connected to some facilities, but here a solution is allowed to leave a client disconnected, at the expenses of paying a penalty. The goal is to minimize the overall sum of connection cost, installation cost, and penalties. The main result of the author is to adapt the best known approximation algorithm for UFL to this generalized setting with penalties, maintaining the same approximation factor. This improves over the previous best known approximation of [Li et al.'15]. In addition, the author links this problem to other interesting network design problems. First, he shows that the same approximation algorithm can be used to approximate another generalization of UFL, where the connection cost of each client is described by a concave nondecreasing function of the distance. Second, he shows that this concave version can be used to model another network design problem, called the Star Inventory Routing problem with Facility Location, introduced by [Jiao and Ravi'19]. As a byproduct of this, the author improves the approximability of this latter problem and also of other capacitated variants. Overall, the approximation bounds developed here substantially improve the existing bounds developed previously in the literature.

Conclusion. The present thesis gives new, solid, and very interesting contributions on a set of fundamental problems in theoretical computer science (TCS). By quantity, the collection of results produced by the author during his PhD program is impressive, far more than enough for a PhD thesis. The quality is also high, as demonstrated by the fact that the author has several publications based on the results of this work, in important TCS conferences. Overall, this is an *excellent* collection of results for a PhD thesis.

Sincerely yours,

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