Abstract

In this thesis we consider several problems from the field of combinatorial optimization. In the first part of the thesis we examine problems related to rank-maximal matching. In the second part we look into the binary variants of the rectangle tiling problem.

In the rank-maximal matching problem we are given a bipartite graph $G = (A \cup P, E)$, in which A denotes applicants, P posts and edges have ranks - an edge (a, p) has rank i if p belongs to (one of) a's ith choices. Ties are allowed, i.e., any applicant a may rank two or more of his neighbours in the same way and then the edges connecting a to these posts have the same rank. A matching M is called *rank-maximal* if the largest possible number of applicants is matched in M to their first choice posts and subject to this condition the largest number of applicants is matched to their second choice posts and so on. It has been previously known that a rank-maximal matching can be computed a in $\mathcal{O}(\min(n, c\sqrt{n})m)$ time, where n denotes the number of vertices, m the number of edges and c the maximal rank of an edge in an optimal solution.

In Section 3 we consider the dynamic version of the problem in which a new applicant or post may be added to the graph and we would like to maintain a rank-maximal matching. We show that after the arrival of one vertex, we are always able to update the existing rank-maximal matching in $\mathcal{O}(\min(c'n, n^2) + m)$ time. All cases of: deletion of a vertex from the graph, addition or deletion of a new edge or even of a change of rank of a given edge can be reduced to the problem of handling the addition of a vertex. We also get an analogous $\mathcal{O}(m)$ result for the dynamic version of the (one-sided) popular matching problem. We have submitted the paper on dynamic rank-maximal matching to *SIAM Journal on Discrete Mathematics (SIDMA).*

In Section 4 we investigate a variant, in which one applicant is manipula-

tive and tries to falsify his preference list so that he has a chance of getting a better post than if he were truthful, i.e. than if he gave a true preference list. We assume that the manipulator knows the preference lists of all the other applicants. We present three strategies corresponding to different criteria of optimization. The paper based on the manipulation strategies of rank-maximal matching has been published at the *24th International Computing and Combinatorics Conference (COCOON 2018).*

In the second part (Section 5) of the thesis we consider various problems related to rectangle tiling. In the most known among them, called RTile , the input is a rectangular array A of nonnegative numbers and a natural number p and the goal is to partition A into at most p rectangular subarrays, called *tiles*, so that the maximal weight of a tile is minimized. The main result that we obtain is a new approximation algorithm for the binary variant of RTile . In the binary variant each element of A is equal to either 0 or 1. The devised algorithm is best possible if we use the only known lower bound. We extend the algorithm for RTile to the dual problem DRTile as well as to their d-dimensional versions. The paper containing the approximation algorithms for the binary RTile and related problems has been submitted to *Information Processing Letters (IPL).*