Report of Thesis of Adam Kunysz

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Overview

This thesis concerns the stable matching problem, with a focus on *strong stability*. Its contribution is composed of three different topics of this theme (Chapters 4-6), drawn from three different papers published in well-known international conferences.

In the stable marriage/roommates problem originally proposed by Gale and Shapley, each person has a *strict* preference over a subset of persons. The goal is to find a stable matching, where no two persons prefer each other to their assigned partners. Irving introduced a very natural extension of this model: the preference is allowed to have ties. However, in this context, the notion of stability has to be properly re-defined. Irving considered three types of stability (depending on how "stable" the matching has to be): super, strong, and weak stability. Roughly speaking, in the first two types of stability, many structures for original stable matching (when there is no tie) carry over and most algorithmic problems can be solved in polynomial time, while the last type of stability notion destroys most of these structures.

Among the three, strong stability is the least understood, and arguably, the most natural definition of stability. Precisely, for a matching to be strongly stable, it cannot have two parties so that one of them strictly prefers the other while the latter is *at least indifferent*. Below is the summary of the three results of the thesis.

Characterisation of Strongly Stable Matching in the Marriage Problem

For the stable marriage problem, Gusfield and Irving have discovered very compact structures to characterise the set of all stable matchings. These structures lead to a variety of algorithmic applications. With strong stability, although it is known that the decision question can be solved in polynomial time, whether analogue structures exist has been left open for a long time—indeed this is one of the twelves open questions in the end of the book of Gusfield and Irving.

The answer is affirmatively answered in this thesis. The author nicely combines a variety of known techniques (in particular the rotation poset of Gusfield and Irving and an algorithm in a paper authored by Kavitha, Melhorn, Michail, and Paluch in 2007) to show that all the known structures can be preserved and discovered in a very efficient manner.

Strong Stability in the Roommates Problem

The decision question of strong stability in the roommates problem was earlier investigated by Scott, who gave an $O(m^2)$ algorithm, where m is the number of edges. The author here gives a new algorithm with $O(nm)$ time, where n is the number of vertices. This algorithm is based on a reduction of the roommates problem to the marriage problem and exploits the previous structural results to discover a strongly stable matching in a fast manner.

In my personal opinion, this algorithm is the most interesting part of the thesis. It gives a totally new perspective on the original algorithm of Irving and establishes a hereto unknown (at least "unnoticed") connection between the marriage and the roommates problem.

The Maximum Weight Strongly Stable Matching in the Marriage Problem

Suppose that edges have weights. A natural objective is to find a stable matching with the maximum weight. When there is no tie, there are two different ways of doing this. The first one is LP-based. Vande Vate showed how to describe the convex hull of all stable matchings by a set of linear inequalities. With such a linear program, one can find the maximum weight stable matching by the Ellipsoid method. The second approach is combinatorial, developed by Gusfield and Irving, where they use their rotation poset characterisation of stable matchings to compute the optimal matching.

The author shows that both approaches can be generalised in the context of strong stability. For the first approach, he modified the stability constraint of the known LP and show that the resulting LP is still integral (hence is again a convex hull of all strongly stable matchings). For the second approach, he shows that the rotation structure he developed in Chapter 4 can again be used in a similar manner as Gusfield and Irving to find the optimal solution.

Overall Evaluation

The author considers a natural optimisation question in the literature from various perspectives. His investigations break new ground, reveal unsuspected connections of known techniques, and answer old open questions. As a result, I believe the author to be well qualified for a Ph.D. degree.

Minor Comment

I give below some comments that I have had when reading the thesis. I hope that they can be useful for the author when he revises the thesis.

- Page 3. 12th last line. "It is rather theoretical".
- Page 6. 2nd last line. "k-th".
- Definition 2.2.5. I think the terminology here is unnatural and counter-intuitive. Shouldn't the weakly blocking pair be called super blocking pair (and vice versa)?
- Page 14. 14th last line. "An F_0-chain".
- Definition 3.1.5. I think the definition is flawed. How about a has $M(a) = M'(a)$?
- Lemma 3.1.8. You say M "dominates" M'. I think the term "dominates" is never defined formally.
- Page 32. Definition of $C(M)$. I would write $(A \cup B) \cap d_M(x) > 0$.
- Page 40. The reference is apparently outdated.
- Page 42. Last line. The notation about the disjoint sets is flawed.

- Page 48. About the LP formulation, there is a new paper by Könemann et al in ORL 2016, giving a very simple proof.
- Page 54. 4th last line before Section 4.3.2. "Rings of sets".
- Page 56. 7th Line. "Lemmas".
- Page 57. Theorem 4.3.5, 3rd item. Are X and X' the equivalent classes? How can $X \subseteq X'$? I believe you mean the P-sets are containing one another.
- Page 61. In the long definition right before Lemma 4.3.10 Why is that the two big unions have asymmetrical subscripts?
- Section 4.5. I think this subsection is the weak part of the thesis. I in fact have difficulty following. The author never gives a clear description of Algorithm 5 (even though the pseudo-code is given and some examples are given). Property 4.5.1, Observation 4.5.2, and Property 4.5.3—I don't know whether they are the properties desired when you design your algorithm, or they are easy consequences of the algorithm.

  Given that this subsection is a crucial part of the thesis, I think it should be revised.
- Reference 52. Should be big O notation.

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