

## Word Equations: Sheet 4

LZ77 on this list is non self-referencing.

**Task 1** Suppose that  $S$  is a length-minimal solution of a word equation  $u = v$ . Let  $w$  be a substring of  $S(u)$ . Show that  $w$  has an occurrence that overlaps with some cut.

*Hint:* Using the inductive definition of the transitive closure may be helpful.

**Task 2** Consider a greedy construction of an LZ77 decomposition , in which the next fragment  $f_i$  is the longest possible substring of  $f_1f_2\cdots f_{i-1}$ . Show that the size of such obtained LZ77 is smallest possible.

**Task 3** Consider a word  $w$  and let  $\ell$  and  $z$  be the sizes of smallest SLP and LZ77 for it. Show that  $\ell \geq z$ . In fact, you do not need to assume that SLP is in Chomsky normal form.

**Task 4** Show that the composition system of size  $n$  can be turned into an SLP of polynomial size. How small you can make the polynomial?