

## Word Equations: Sheet 7

**Task 1** Show that the approximation algorithm for the smallest grammar problem using the recompression approach returns an SLP of size  $\mathcal{O}(g' \log n)$ , where  $g'$  is the size of the smallest grammar in which we additionally allow rules of the form  $A \rightarrow B^\ell$ , that have size 1.

**Task 2** Let SLP  $\mathcal{G}$  contain no production  $X \rightarrow \alpha$  with  $|\alpha| \leq 1$  and assume that every production is used in the derivation defined by  $\mathcal{G}$ . Then  $|\mathcal{G}| \leq 2|\text{val}(\mathcal{G})| - 1$ .

**Task 3** Show that the computation of SLP by the recompression based approach can be divided into two phases, in first the issued credit is  $\mathcal{O}(g + g \log(n/g))$  and in the second the representation cost is at most  $\mathcal{O}(g)$ .

*Hint*<sup>1</sup>

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<sup>1</sup>For the second part, using the exercise above. For the first, try to find appropriate moment in the computation so that it fits the later analysis.