Conjunctive grammars generate non-regular unary languages

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August 21, 2007

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- Intuition of the semantics:
  - w is derived such production iff it is derived by each α_i
  - w is derived from $\alpha_i = N_1 \cdot N_2 \cdot \ldots \cdot N_k$ iff $w = w_1 w_2 \ldots w_k$ and w_j is derived from N_j for each j

Example

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$$\Sigma = \{a, b, c\},$$
$$N = \{S, B, C, E, A\}$$

$$S \to (AE)\&(BC)$$
$$A \to aA|\epsilon$$
$$B \to aBb|\epsilon$$
$$C \to cC|\epsilon$$
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$S \to (AE)\&(BC)$	$\{a^n b^n c^n : n \in \mathbb{N}\}$
$A \to aA \vert \epsilon$	a^*
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- from possible extensions of CFG this keeps the meaning of language equations
- good parsing properties

Formal syntax

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A conjunctive grammar is a $\langle \Sigma, N, S, P \rangle$ where

- Σ is a finite alphabet
- N—set of non-terminal symbols
- S—starting symbol
- P—set of productions of a form

$$A \to \alpha_1 \& \alpha_2 \& \ldots \& \alpha_k , \quad \alpha_i \in (\Sigma \cup N)^*$$

Rewriting

Semantics

By *term rewriting*.

Rewriting

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Generalizes the Chomsky rewriting.

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Drawbacks

- There are more generalizations.
- Slightly problematic to handle.

Language equations

Semantics

With each nonterminal A we associate a language L_A.

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Remark

In the CFG case the only allowed operations are $\cup$ and $\cdot$.

Example revisited

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$$\begin{aligned} S &\to (AE)\&(BC) & L_S &= (L_A \cdot L_E) \cap (L_B \cdot L_C) \\ A &\to aA|\epsilon & L_A &= \{a\} \cdot L_A \cup \{\epsilon\} \\ B &\to aBb|\epsilon & L_B &= \{a\} \cdot L_B \cdot \{b\} \cup \{\epsilon\} \\ C &\to cC|\epsilon & L_C &= \{c\} \cdot L_C \cup \{\epsilon\} \\ E &\to bEc|\epsilon & L_E &= \{b\} \cdot L_E \cdot \{c\} \cup \{\epsilon\} \end{aligned}$$

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Negative results

- Mainly open questions

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Do all conjunctive grammars over unary *alphabet generate only* regular *languages?*

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This should be true since regular sets are closed under

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Yes

Intuition

This should be true since regular sets are closed under

- *concatenation*
- *intersection*
- *union*

Result

Theorem (Disproving the conjecture)

Conjunctive grammars generate non-regular languages over unary alphabet.

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Theorem (Extension)

For every regular language $R \subseteq \{0, 1, \ldots, k-1\}^*$ language

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is a unary conjunctive language.

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For every regular language $R \subseteq \{0, 1, \ldots, k-1\}^*$ language

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is a unary conjunctive language. Positional notation.

Language

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We identify a^n with n and work with sets of integers.

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Solution

$$\begin{aligned}
L_1 &= \{1 \cdot 4^n : n \in \mathbb{N}\}\,, \\
L_2 &= \{2 \cdot 4^n : n \in \mathbb{N}\}\,, \\
L_3 &= \{3 \cdot 4^n : n \in \mathbb{N}\}\,, \\
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\begin{aligned}
B_1 &= (B_2 B_2 \cap B_1 B_3) \cup \{1\}, \\
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This effectively manipulates the positional notation.

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Example

For example L_1, the rule is

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So we want to prove that

$$L_1 = (L_2L_2 \cap L_1L_3) \cup \{1\}$$

Details–what is in $B_2 B_2$

Proof.

What are the possible non-zero symbols in $B_2 B_2$?

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Either only 1 or $\{2, 2\}$. $\square$

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Either only 1 or $\{1, 3\}$.
We compare this with 1 or $\{2, 2\}$ from $B_2 B_2$.
The only possibility is only 1. $\square$

First step: $ij0^*$

Theorem

For every k and $0 < i, j < k$ languages

$$\{a^n : \exists w \in i0^* \ w \text{ read as a number is } n\}$$
$$\{a^n : \exists w \in ij0^* \ w \text{ read as a number is } n\}$$

are unary conjunctive languages.

Idea

Done in the same way.

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- *Intersections of concatenations filter out wrong combination of leading symbols.*

Second step: any regular language

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For every k and $R \subset \{0, \ldots, k-1\}^*$

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Let $\langle\{0, \ldots, , k-1\}, Q, q_0, F, \delta\rangle$ recognizes R.

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Information the indices carry:

- *leading symbol i*
- *second leading symbol j*
- *q—the computation of M on the rest of the word*

Productions for $B_{i,j,q}$

Example

$$B_{i,j,q} \to \left(\&_{n=1}^{4} B_{i-1,j+n} B_{k-n,x,q'} \right)$$

where x, q' such that $q \in \delta(q', x)$

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where x, q' such that $q \in \delta(q', x)$

$$\begin{array}{ccccc}
 & & k-n & x & \overbrace{\ldots}^{\text{state } q'} \\
+ & i-1 & j+n & \multicolumn{2}{c}{00\ldots0} \\
\hline
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In particular it generates non-regular languages.
We effectively manipulate positional notation.

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- Unambiguity of the language. The construction for $R = ij0^*$ can be made unambiguous. What happens in general?

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Related topics and following work

- Unambiguity of the language. The construction for $R = ij0^*$ can be made unambiguous. What happens in general?
- The result can be extended to a larger class of languages [A. Jez, A. Okhotin, CSR 2007].
- Instead of grammars we can focus on sets of integers. Equations on sets of integers using $\cap$, $\cup$ and $+$ defined as

$$A + B = \{a + b : a \in A, b \in B\} .$$

[A. Jez, A. Okhotin, TALE 2007].

Open questions

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  - better recognition (space/time)
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- Unambiguity of the constructed unary languages
- Closure under complementation in the unary case.