1 Quadratic Assignment Problem

In the Quadratic Assignment Problem (QAP) of size $n \in \mathbb{N}$, a set of n facilities must be assigned to a set of n locations with minimization of the assignment cost defined by a function $c : \Pi \to \mathbb{R}$ which returns a cost $c(\mathbf{p}) \in \mathbb{R}$ for an assignment $\mathbf{p} \in \Pi$. An assignment of facilities to locations $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is a permutation of the set of locations $\{1, 2, \ldots, n\}$, where each element p_i corresponds to the location assigned to the i-th facility, and consequently the set of assignments Π is the set of permutations of size n.

The cost function is defined by a flow matrix $\mathbf{F} \in \mathbb{R}^{n \times n}$, where each element f_{ij} corresponds to the flow between i-th and j-th facilities, and a distance matrix $\mathbf{D} \in \mathbb{R}^{n \times n}$, where each element d_{ij} corresponds to the distance between i-th and j-th locations, in the following way:

$$c(\mathbf{p}) = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} d_{p_i p_j}. \tag{1}$$

2 Quadratic Three-Dimensional Assignment Problem

The first definition of the quadratic three-dimensional assignment problem (Q3AP) was introduced by William P. Pierskalla. It is usually formulated as a minimization problem

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{p=1}^{n} b_{ijp} x_{ijp} + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{p=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{q=1}^{n} c_{ijpklq} x_{ijp} x_{klq}, \tag{2}$$

where $x_{ijp} \in \{0, 1\}$ and

$$\sum_{j=1}^{n} \sum_{p=1}^{n} x_{ijp} = 1, \qquad \sum_{i=1}^{n} \sum_{p=1}^{n} x_{ijp} = 1, \qquad \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijp} = 1, \tag{3}$$

with given $n \in \mathbb{N}$, $b_{ijp} \in \mathbb{R}$ and $c_{ijpklq} \in \mathbb{R}$.

3 Quadratic Three-Dimensional Assignment Problem Reformulated

Popular benchmark instances for QAP, such as the classic QAP instances stored in the QAPLIB repository, may be used to create benchmark instances for Q3AP in such a way that the same flow and distance matrices, A and B, are used and the objective function is

$$F(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{p_i p_j} \cdot A_{q_i q_j} \cdot B_{ij}^2. \tag{4}$$