Introduction to Flight Simulation (List 2)

Date: 16.10.2012

1. Consider the following Butcher tableau:

0				
$\frac{1}{3}$	$\frac{1}{3}$			
1	$\frac{1}{4}$	$\frac{3}{4}$		
1	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$	
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

(It is just an example. It is not supposed to give a good order.)

   (a) Write out the complete method, using k_1, k_2, k_3, k_4 and assuming function $\overline{F}$.

   (b) When the function $\overline{F}$ has form $\lambda x : \mathcal{R} \quad \overline{y} : \mathcal{R}^n \quad (1, \overline{G}(x, \overline{y})\ )$, then the value of the first component of $\overline{F}$ can be easily predicted. What is the value? This fact can be used, to completely eliminate $\overline{F}$ from the computation of the approximation.

   Do this.

   (c) Do the same with the standard Runge Kutta method RK41. The result is the Runge-Kutta method that you usually find in text books.

2. Suppose that we have a weight with mass m hanging on a rope of length l in gravity g. Suppose that the rope is connected in the origine of a three dimensional coordinate system.

   You may assume that the weight is a point, and that the rope has no mass.

   You may assume that the rope is always tense, and doesn't break.

   (a) Give a function $f(x_1, x_2)$ that expresses the x_3-coordinate of the position of the mass in terms of x_1 and x_2. When is $f(x_1, x_2)$ defined?

   (b) Give a function $\overline{F}(x_1, x_2)$ that specifies the force on the mass as a three dimensional vector.

   (c) Give a differential equation that specifies the movements of the weight on the rope.

3. Same task as in the previous question, but now the rope is elastic. We assume that its force is equal to $F = (l - l_e).c_e$, where l is the actual length of the elastic, l_e is the length when the elastic is not stretched, and c is a constant that determines how much force depends on stretching. (The elasticity constant.)

   You may assume that the elastic is always stretched.