Introduction to Flight Simulation (List 1)

Deadline: 19.10.2016

1. Consider the second order differential equation $y'' + y = 0$.
   (a) Prove that the function $y = \sin(x)$ and $y = \cos(x)$ are solutions of this differential equation.
   (b) Prove that if some function $y = f(x)$ is a solution of the differential equation, then the function $y = \lambda f(x)$ is also a solution, for arbitrary $\lambda \in \mathcal{R}$.
   (c) Prove that if two functions $y = f_1(x)$ and $y = f_2(x)$ are solutions of the differential equation, then the function $y = f_1(x) + f_2(x)$ is also a solution.
   (d) At this point, you can combine **a,b,c** to characterize all solutions.
   (e) Which solution has $y(0) = 0, \quad y'(0) = 1$?

2. Consider a general second order, linear differential equation $c_2 y'' + c_1 y' + c_0 y = 0$, where c_0, c_1, c_2 are constants.
   (a) Show that if a is a zero of the polynomial $c_2 x^2 + c_1 x + c_0 = 0$, then $y = e^{ax}$ is a solution of the differential equation. The polynomial is called *the characteristic polynomial* of the differential equation.
   (b) Show that if the characteristic polynomial has has form $c_2 x^2 + c_1 x + c_0 = c_2 (x - d)^2$, that then both $y = e^{dx}$ and $y = x e^{dx}$ are solutions.
   (c) Show that if the differential equation has two solutions $f_1(x)$ and $f_2(x)$, then $\lambda_1 f_1(x) + \lambda_2 f_2(x)$ is also a solution, for arbitrary $\lambda_1, \lambda_2 \in \mathcal{R}$.

   (The differential equation in Task 1 had form $y'' + y = 0$, which means that its characteristic polynomial has form $x^2 + 1 = 0$. This polynomial has two zeros, namely i and $-i$. As a consequence, there are two solutions

   $$y = e^{ix}, \qquad y = e^{-ix}.$$

   We have $e^{ix} = \cos x + i \sin x$, and $e^{-ix} = \cos x - i \sin x$, These two solutions can be combined using $\lambda_1 = \lambda_2 = \frac{1}{2}$, and $\lambda_1 = -\frac{i}{2}, \lambda_2 = \frac{i}{2}$. This gives the solutions that were given in Task 1a)

3. Find all solutions of

$$y'' + y' - 6y = 0.$$

4. Find all real valued solutions of

$$y'' - 2y' + 2y = 0.$$