Flight Simulation (List 3)

Deadline: 16.11.2016

1. (a) What is the norm of the following quaternions:
   a (1; 2, 3, 4)
   b (-1; 3, -2, 1)
   c (1; 1,1,1)
   d (2; 2,1,-3)

   (b) Assume that the quaternions represent rotations. For each of the quaternions, what is the rotation axis, and what is the rotation angle? In general, there are no exact expressions for the angle, so just use a calculator. (Please give the angles in degrees.)

   (c) Apply the rotations a,b,c,d on the vector $(2, -1, 1)$.

   (d) Give the rotation matrices that correspond to the quaternions.

   (e) What is $a.b$? (Quaternion product?) What is $b.c$? Is it equal to $c.b$?

2. Consider a Rubik cube. It has 6 sides, called **L**eft, **R**ight, **U**p, **D**own, **F**ront, **B**ack.

   The sequence

   $$(R \cdot D^{-1} \cdot R^{-1} \cdot D)^2 \cdot U \cdot (D^{-1} \cdot R \cdot D \cdot R^{-1})^2 \cdot U^{-1}$$

   rotates two corners. R denotes: Rotating the right side in clock direction. D^{-1} denotes rotating the down side in anti-clock direction.

   Let's assume that the cube stands in front of us, oriented in such a way that the right side points in positive X-direction, top points in positive Z-direction, and back points in positive Y-direction.

   Give for each of the rotations $R, R^{-1}, L, L^{-1}, U, U^{-1}, D, D^{-1}, F, F^{-1}, B, B^{-1}$ the quaternion.

   Compute the quaternion $L.U$. (First U, then L.) Also give the quaternion $B.R$. (First R, then B.) (Note: You can use the quaternion finder that I gave to you, if you are willing to do some assembly work.)

3. Let $\overline{e}$ be a vector of unit length. Let φ be a real number. We want to compute the effect of rotating vector $\overline{x}$ around axis $\overline{e}$ with angle φ. First

define $\overline{\omega} = \overline{e}\varphi$. Then define $\overline{F}(\overline{x}) =$

$$\overline{x}+(\overline{\omega}\times\overline{x})+\frac{\overline{\omega}\times(\overline{\omega}\times\overline{x})}{2!}+\frac{\overline{\omega}\times(\overline{\omega}\times(\overline{\omega}\times\overline{x}))}{3!}+\frac{\overline{\omega}\times(\overline{\omega}\times(\overline{\omega}\times(\overline{\omega}\times\overline{x})))}{4!}+\cdots$$

Prove that $\overline{F}(\overline{x})$ is indeed a rotation around axis $\overline{e}$ over angle φ. (The proof is not difficult. If your proof is more than half a page long, then something is going wrong.)

Why is this method less practical than using quaternions?

4. Consider a cube with vertices

A	(0,0,0)
B	(1,0,0)
C	(1,1,0)
D	(0,1,0)
E	(0,0,1)
F	(1,0,1)
G	(1,1,1)
H	(0,1,1)

Its edges are AB, AD, AE, CB, CD, CG, FE, FB, FG, HE, HD, HG.

We start by looking at this cube from the direction of negative Y-axis. Then positive X will point to the right, and positive Z will point upwards. Because this is a boring perspective, we **(1)** First rotate the cube along $\phi = 30\,\text{deg}$ around the Z-axis, and after that **(2)** rotate the cube by $\theta = 15\,\text{deg}$ around the X-axis,

(a) Give the quaternions q_1 and q_2 for these rotations. Give $q_2.q_1$. (The numbers are not nice. Use a pocket calculator.)

(b) We need to further rotate the cube into camera coordinates. That means that positive Z will be behind us, positive X points will point to the right, and positive Y will point upwards.

The quaternion that belongs to this rotation is $q_3 = (1;\ 1, 0, 0)$. Compute $q_3.q_2.q_1$.

(c) Give the rotation matrix of $q_3.q_2.q_1$. Again use a pocket calculator.

(d) We can apply the rotation on the cube above. Give the transformed coordinates.

(e) We can now draw the cube in an orthogonal projection, simply by ignoring the Z-coordinates. Do this.

(f) In order to get a perspective (frustum) projection, we move our camera a little bit backwards, so that the cube is completely in front of us (at negative Z positions). This is equivalent to applying the transformation $f(x, y, z) = (x, y, z - 3)$ on the vertices of the cube.

Perspective projection can be obtained by applying $f(x, y, z) = (-\frac{x}{z}, -\frac{y}{z})$. On the points of the cube. Compute and draw the result.