# Interactive Proof Tools
## Assignment 6

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INFORMATIK

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`http://www.mpi-sb.mpg.de/~nivelle/teaching/intprooftools2003/main.html`

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The exercises 6.1 to 6.3 belong to our project of formalizing a (propositional) resolution calculus in PVS.

**Exercise 6.1** A (propositional) *clause* is a finite disjunction of literals, where a *literal* is either a propositional variable or the negation of a propositional variable.

1. Formalize clauses in PVS, i. e., define non-empty types of propositional variables, literals and clauses.
   **Hint:** A clause can be represented as a multiset of literals.

2. Define a function `occurs` which maps a set of clauses to the set of its propositional variables.
   **Hint:** First define a function which maps a single clause to the set of its variables.

3. Prove that `occurs` maps a finite set of clauses to a finite set of variables.

**Exercise 6.2** An *interpretation* is a mapping of propositional variables to *truth values*.

1. Define a type of truth values and a type of interpretations.

2. Define a predicate `true?` which holds of a clause and an interpretation iff the clause evaluates to true in the interpretation.

3. Define predicates `valid?` and `satisfiable?` which hold of a clause if it is *valid* (true in all interpretations) or *satisfiable* (true in at least one interpretation), respectively.

4. Prove that the clauses $A \vee \neg A$ resp. $A \vee \neg B$ (with propositional variables $A$, $B$) are valid resp. satisfiable.

5. Extend the definitions of validity and satisfiability to sets of clauses. Prove that validity of a set of clauses implies satisfiability.

**Exercise 6.3** Prove the following theorem: Given a clause $c$ and two interpretations $I_1$ and $I_2$, if $I_1$ and $I_2$ agree on all propositional variables occuring in $c$ then $c$ is true in $I_1$ iff $c$ is true in $I_2$.

**Exercise 6.4** Prove the following formulas in Coq:

1. `(x, y, z: nat) (le x y) /\ (le y z) -> (le x z)`

2. `(x, y: nat) (le x y) \/ (le y x)`