Interactive Proof Tools
Assignment 9

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INFORMATIK

`http://www.mpi-sb.mpg.de/~nivelle/teaching/intprooftools2003/main.html`

The exercises 9.1 and 9.2 belong to our project of formalizing and verifying a propositional resolution calculus in PVS.

Exercise 9.1 Prove the Interpretation Existence Theorem (slide 6) in PVS.
Hint: Follow the proof sketch on the slides. Don't try to prove the theorem in one step. Rather, most of the statements in the proof sketch should be singled out as lemmas and proven separately.

Exercise 9.2 Prove the following lemmas which may be useful in the proof of the Interpretation Existence Theorem.

```
C: VAR clause
S, T: VAR finite_set[symbol]
Gamma: VAR set[clause]

cand_int_mono2: LEMMA
  subset?(S,T) AND subset?(occurs(C), S) AND
    NOT true?(mkInterpret(cand_int(Gamma,S)), C) IMPLIES
      NOT true?(mkInterpret(cand_int(Gamma,T)), C)

consequences_occurs: LEMMA
  occurs(consequences(Gamma)) = occurs(Gamma)
```

Exercise 9.3 Assume $X : \mathrm{Set}$. Define the functions

1. $\mathrm{length} : \mathrm{List}\, X \to \mathrm{Nat}$,
2. $\mathrm{append} : \mathrm{List}\, X \to \mathrm{List}\, X \to \mathrm{List}\, X$, and
3. $\mathrm{reverse} : \mathrm{List}\, X \to \mathrm{List}\, X$

with their usual meaning through recursion over lists.
Use the recursion operator

$$\mathrm{rec}_{\mathrm{List}} : \prod X{:}\,\mathrm{Set} \prod S{:}\,\mathrm{Set}\; S \to (X \to \mathrm{List}\, X \to S \to S) \to \mathrm{List}\, X \to S.$$

The $\mathrm{rec}_{\mathrm{List}}$ reduction rules are

$$(\mathrm{rec}_{\mathrm{List}}\, X\, S\, f\, g\, \mathrm{nil}) \Rightarrow f$$
$$(\mathrm{rec}_{\mathrm{List}}\, X\, S\, f\, g\, (\mathrm{cons}\, x\, xs)) \Rightarrow (g\, x\, xs\, (\mathrm{rec}_{\mathrm{List}}\, X\, S\, f\, g\, xs))$$