Exercises for Friday 2nd of November 11.15

October 30, 2001

1. Complete the sequent calculus mentioned on Page 5,6,7 of the slides on Sequent Calculus.

2. Prove the following formulae in intuitionistic natural deduction.

   (a) $(A \vee A) \leftrightarrow (A \vee \bot)$

   (b) $(B \wedge A) \leftrightarrow ((A \wedge B) \wedge A)$.

   (c) $(A \rightarrow B) \rightarrow \neg(A \wedge \neg B)$.

   (d) $\neg(A \wedge \neg A)$.

   (e) $(\neg\neg\neg A) \rightarrow \neg A$.

   (f) $\exists x{:}X\ (P(x) \wedge Q(x)\ ) \leftrightarrow (\exists x{:}X\ P(x)\ ) \vee (\exists x{:}X\ Q(x)\ )$.

   (g) $(A \rightarrow B \rightarrow C) \leftrightarrow (A \wedge B \rightarrow C)$.

   (h) $x + s(y) \approx s(y) + x$ from premisses
   $x + y \approx y + x$
   $\forall x, y{:}\mathrm{Nat}\ x + s(y) \approx s(x + y)$,
   $\forall x, y{:}\mathrm{Nat}\ s(x) + y \approx s(x + y)$.

3. Give proofs for the following formulae in Classical Sequent Calculus.

   (a) $A \vee (A \rightarrow B)$

   (b) $(A \rightarrow B) \leftrightarrow (\neg A \vee B)$.

   (c) $\exists x{:}X\ \neg P(x) \vee \forall x{:}X\ P(x)$.

   (d) $(\exists y{:}Y\ \forall x{:}X\ P(x, y)\ ) \rightarrow \forall x{:}X\ \exists y{:}Y\ P(x, y)$.

   (e) $(\forall x{:}X\ P(x) \vee \forall x{:}X\ Q(x)\ ) \rightarrow (\forall x{:}X\ P(x) \vee Q(x))$