

Exercises due on Friday 23d of November 11.15

November 16, 2001

1. Expand the following formulae, inserting the \dots 's that have been omitted:

- (a) $(f\ x)$,
- (b) $(f\ a\ b\ c)$,
- (c) $(f\ (g\ a\ b)\ (g\ b\ a))$,
- (d) $(s\ (s\ (s\ 0)))$.

Expand the following formulae, using Π instead of \Rightarrow :

- (a) $X \Rightarrow Y \Rightarrow Z$,
- (b) $(\text{List Nat}) \Rightarrow (\text{List Nat}) \Rightarrow (\text{List Nat})$.
(also expand the \cdot)

2. Which of the following pairs of terms are $\alpha, \beta, \delta, \eta$ -equivalent?

- (a) $\lambda x: X (p\ x)$ and $\lambda y: X (q\ x)$,
- (b) $\lambda p: (X \Rightarrow \text{Form}) \lambda x: X (p\ x)$ and $\lambda q: (X \Rightarrow \text{Form}) \lambda y: X (q\ y)$,
- (c) $\lambda x: X (f\ x)$ and $(\lambda y: (X \Rightarrow X) y) \cdot f$,
- (d)

true

and

$(K\ (K\ \text{true})) \cdot \text{false}$,

in the context

$\text{Bool: Type, true: Bool, false: Bool}$,

$K := \lambda x: \text{Bool } \lambda y: \text{Bool } x: (\text{Bool} \Rightarrow \text{Bool})$.

3. Derive the following type judgments, using the typing rules presented in the slides:

- (a) $\text{Nat: Type, } 0: \text{Nat, } s: \text{Nat} \Rightarrow \text{Nat} \vdash (s\ (s\ (s\ 0))): \text{Nat}$,
- (b) $\text{Nat: Type, } 0: \text{Nat, } s: \text{Nat} \Rightarrow \text{Nat}, \vdash \lambda x: \text{Nat } (s\ x): (\text{Nat} \Rightarrow \text{Nat})$,
- (c) $\text{Bool: Type, true: Bool, nil: } \Pi X: \text{Type } (\text{List } X)$,
 $\text{cons: } (\Pi X: \text{Type } X \Rightarrow (\text{List } X) \Rightarrow (\text{List } X)) \vdash$
 $(\text{cons Bool true } (\text{cons Bool false } (\text{nil Bool}))) : (\text{List Bool})$.