# 1 Exercises for the Lecture Automated Theorem Proving

(To be completed by May 24th, 18.00)

1. Consider the clause set $S =$

| | |
|---|---|
| $[P_1, Q_1, A]$ | 1 |
| $[P_1, \neg Q_1]$ | 2 |
| $[\neg P_1, Q_1]$ | 3 |
| $[\neg P_1, \neg Q_1]$ | 4 |
| $[P_2, Q_2, \neg A]$ | 5 |
| $[P_2, \neg Q_2]$ | 6 |
| $[\neg P_2, Q_2]$ | 7 |
| $[\neg P_2, \neg Q_2]$ | 8 |

   Give all possible resolvents that can be obtained in one iteration of resolution from $S$. (Do not forget that clauses are multisets)

2. Next consider the $A$-order $\succ$ defined by

$$P_1 \succ P_2 \succ A \succ Q_1 \succ Q_2.$$

   Give all possible resolvents that can be obtained in one iteration of $A$-ordered resolution from $S$.

3. Consider the selection function $\Sigma$, defined by $\Sigma(c) = \{L \mid L \in c \text{ and } L \text{ is negative }\}$.

   Construct all possible resolvents that can be obtained in one iteration of $A$-ordered resolution with selection.

4. Construct a complete refutation of $S$, using $A$-ordered resolution with selection, and applying subsumption as much as possible.

5. Give an example of set of clauses that cannot be refuted without factoring.

6. A clause that contains a complementary pair $\neg A, A$ is called a *tautology*. Prove the following:

   (a) Let $c$ be a tautology, let $c'$ be a factor of $c$. Then $c'$ is a a tautology.

   (b) Let $c$ be a tautology, let $c'$ be an arbitrary clause. Let $d$ be a resolvent of $c$ and $c'$. Then $c'$ subsumes $d$.

   It can be concluded that tautologies can be eliminated without loss of completeness. Identify the tautologies in the refutation constructed in exercise (4).