1 Exercises for the Lecture Automated Theorem Proving

(To be completed by May 31st, 18.00)

1. Let the A-order $\succ$ be defined by:

   $$P_1 \succ P_2 \succ A \succ Q_1 \succ Q_2.$$

   Let the selection function Σ be defined by:

   $$\Sigma(c) = \neg Q_1 \text{ if } (\neg Q_1) \in c, \quad \emptyset \text{ otherwise.}$$

   Consider the clause set $S =$

$[P_1, \neg Q_1]$	1
$[\neg P_1, Q_1]$	2
$[\neg P_1, \neg Q_1]$	3
$[P_2, Q_2, \neg A]$	4
$[P_2, \neg Q_2]$	5
$[\neg P_2, Q_2]$	6
$[\neg P_2, \neg Q_2]$	7
$[\neg A, Q_2]$	8
$[\neg Q_2, \neg Q_2]$	9
$[Q_2, \neg Q_2]$	10
$[\neg Q_2]$	11

   Convince yourself of the fact that S is a saturated set, and that it is a saturation of the clauses $1, \ldots, 7$.

   (Actually, S would already have been a saturated set without 9 and 10. Why?)

2. Rank the clauses of S.

3. Perform the model construction for S.

4. Show that for the ranking function the following holds: If c_1 is ranked before c_2, and c_2 is ranked before c_3, then c_1 is ranked before c_3. (This was implicitly used in the completeness proof. Without it, the ranking function would be useless as a ranking function)

5. Show that: If c_1 is ranked before c_2, then $c_1 \cup d$ is ranked before $c_2 \cup d$. ($\cup$ denotes multiset union)

6. (The following fact was used in the completeness proof, but not really proven)

Let $R_1 \cup R_2$ be the resolvent of $[\neg A] \cup R_1$ with $[A] \cup R_2$. Assume that $A \succ R_2$. Show that $R_1 \cup R_2$ has a lower rank than $[\neg A] \cup R_1$. (Decompose the problem, s.t. the claims in the two previous questions can be used)

7. Given the A-order of the first question, do $[\neg P_1], [P_1]$ make $[P_2]$ redundant?

   Do $[\neg P_2], [P_2]$ make $[P_1]$ redundant?

   Does $[\,]$ make $[P_1, \neg Q_1]$ redundant?

   Do $[P_2, Q_2, \neg A]$, $[P_2, \neg Q_2]$, $[\neg P_2, Q_2]$, $[\neg P_2, \neg Q_2]$ make $[\neg A]$ redundant? And $[\neg A, \neg A]$?