1 Exercises for the Lecture Automated Theorem Proving

(To be completed by June 14th, 18.00)

1. Determine whether the following pairs of formulas have unifiers. If they have, then construct a most general unifier.

$p(X, Y, Z)$	$p(Y, Z, X)$
$p(X, Y)$	$p(X, Y, Z)$
$p(X)$	$q(Y)$
$p(X, X)$	$p(s(0), s(Y))$
$p(X, X)$	$p(s(Y), s(Z))$
$p(X, X)$	$p(s(0), s(1))$
$p(s(X), X)$	$p(s(Y), Y)$
$p(s(X), X)$	$p(Y, Y)$
$p(s(X), X)$	$p(t(Y), Y)$

2. Consider the clause $[\neg p(X), p(s(X))]$, which can resolve with itself as follows:

$$\frac{[\neg p(X), p(s(X))] \qquad\qquad [\neg p(Y), p(s(Y))]}{[\neg p(X), p(s(s(X)))]}$$

   The following clause set S is a set of instances of $[\neg p(X), p(s(X))]$.

   $[\neg p(0), p(s(0))]$,
   $[\neg p(s(0)), p(s(s(0)))]$,
   $[\neg p(s(s(0))), p(s(s(s(0))))]$,
   $[\neg p(s(s(s(0)))), p(s(s(s(s(0)))))]$,
   $[\neg p(a), p(s(a))]$,
   $[\neg p(s(a)), p(s(a))]$,

   Construct all clauses that are derivable by resolution from S in one step. Give for each clause the substitution Θ with which it is an instance of $[\neg p(X), p(s(s(X)))]$. (Such substitution always exists, due to the lifting theorem)

3. Consider the following clause set:

1	$[p(0), p(s(0))]$	(init)
2	$[p(0), \neg p(s(0))]$	(init)
3	$[\neg p(0), p(s(0))]$	(init)
4	$[\neg p(0), \neg p(s(0))]$	(init)

   which (for example) has the following refutation:

5	$[p(0), p(0)]$	res 1,2
6	$[p(0)]$	fact 5
7	$[p(s(0))]$	res 3,6
8	$[\neg p(s(0))]$	res 4,6
9	$[\,]$	res 7,8

Consider the following set of non-ground clauses:

A	$[p(X), p(s(X))]$
B	$[\neg p(X), p(Y)]$
C	$[\neg p(X), \neg p(s(X))]$

For every clause c in 1,2,3,4, there is a clause d in A, B, C s.t. c is an instance of d. Lift the refutation 5,6,7,8,9 to a resolution refutation of $A, B.C$.

4. Consider the A-order $\succ$ defined by $A_1 \succ A_2$ iff $\#A_1 > \#A_2$. Here $\#A$ denotes the number of symbols in A. So $\#p(a) = 2$, and $\#p(s(a)) = 3$. Construct a refutation of $1, 2, 3, 4$ (from the previous question) that is allowed by $\succ$.

   Lift this refutation to a resolution refutation of A, B, C.

5. Given the A-order of the previous question, which of the following resolvents have to be constructed:

$[p(s(0)), q(0)]$	with	$[\neg p(s(0)), q(0)]$
$[p(0), q(s(0))]$	with	$[\neg p(X)]$
$[p(s(X)), q(X)]$	with	$[\neg p(s(0)), q(0)]$
$[p(X, Y), q(X, X)]$	with	$[\neg p(s(s(Z)), Z)]$
$[\neg p(X), p(s(X))]$	with	$[\neg p(Y), p(s(Y))]$
$[\neg p(X), p(s(Y))]$	with	$[\neg p(Z), p(s(Z))]$
$[\neg p(X), p(s(X))]$	with	$[p(a)]$
$[\neg p(X), p(s(X))]$	with	$[\neg p(s(s(Y)))]$

   In case, the resolvent needs to be constructed, give an instance of the resolution step that is allowed by $\succ$.