Theorem Proving: (List 1)

Deadline: 02.03.2016

1. Prove, using one-sided sequent calculus, the following formulas:
   (a) $\neg(A \land B) \to (\neg A \lor \neg B)$,
   (b) $(\neg A \lor \neg B) \to \neg(A \land B)$,
   (c) $A \leftrightarrow \neg\neg A$,
   (d) $(A \to B) \to (B \to C) \to (A \to C)$,
   (e) $((A \land A) \to B) \to (A \to B)$,
   (f) $(A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow \neg B)$
2. Give the resuls of the substitutions below:
   (a) $f(g(x), x)[x := g(y)]$
   (b) $f(g(x), y)[x := g(y)]$
   (c) $p(f(y, x), g(x))[y := x]$
   (d) $(\ \forall y\ p(f(y, x), g(y))\ )\ [y := x]$
   (e) $(\ \forall y\ p(f(y, x), x)\ )[x := y]$
   (f) $(\ \forall y\ (\ p(f(z, g(y)), g(x))\ ) \lor (\exists z\ g(z) \approx y)\ )\ [x := g(y)]$,
   (g) $\forall y\ f(x, y) \approx x \to \forall x\ (\ f(x, y) \approx x\ )\ [y := g(y)][x := g(z)]$
3. Prove the following formulas using one-sided sequent calculus:
   (a) $\forall x\ x \approx x$
   (b) $\forall x\ s(x) \approx x \vdash \forall x\ s(s(s(x))) \approx x$
   (c) $\forall x\ (\ P \to Q(x)\ ) \vdash P \to \forall x\ Q(x)$
   (d) $\forall x\ (P(x) \land Q\ ) \vdash (\forall x\ P(x)\ ) \land Q$
   (e) $(\neg \forall x\ P(x)) \leftrightarrow \exists x\ \neg P(x)$
   (f) $\exists x\ (\ D(x) \to \forall y\ D(y)\ )$ This the famous drinker paradox, which reveals the crazyness of classical logic.
   (g) $\forall x\ f(x) \approx x,\ \ \forall x \exists y\ p(f(x), y) \vdash \forall x \exists y\ p(x, f(y))$

(h)

$$\forall x \forall y\ [\ R(x,y) \rightarrow \exists z\ [\ R(x,z) \wedge R(z,y)\ ]]$$
$$\neg \exists x\ R(x,x)$$
$$\vdash$$
$$\forall x \forall y\ [\ R(x,y) \rightarrow \exists z\ [\ z \not\approx x \wedge z \not\approx y\ ]]$$

(i)

$$\forall x \forall y\ [(\ E(x) \wedge E(y)\ ) \rightarrow \exists z\ [\ E(z) \wedge S(z,x,y)\ ]]$$
$$\forall x \forall y\ [\ (\neg E(x) \wedge \neg E(y)) \rightarrow \exists z\ [\ E(z) \wedge S(z,x,y)\ ]]$$
$$\forall x \forall y \forall v \forall w\ [\ (\ S(v,x,y) \wedge S(w,x,y)\ ) \rightarrow v \approx w\ ]$$
$$\vdash$$
$$\forall x \exists y\ [\ E(y) \wedge \forall z\ [\ S(z,x,x) \rightarrow y \approx z\ ]]$$