Theorem Proving:List 2

Deadline: 09.03.2016

1. Determine whether the following pairs of formulas have unifiers. If they have, then construct a most general unifier.

$p(X,Y,Z)$	$p(Y,Z,X)$
$p(X,Y)$	$p(X,Y,Z)$
$p(X)$	$q(Y)$
$p(X,X)$	$p(s(0),s(Y))$
$p(X,X)$	$p(s(Y),s(Z))$
$p(X,X)$	$p(s(0),s(1))$
$p(s(X),X)$	$p(s(Y),Y)$
$p(s(X),X)$	$p(Y,Y)$
$p(s(X),X)$	$p(t(Y),Y)$

2. Consider the clause $[\ \neg p(X), p(s(X))\ ]$, which can resolve with itself as follows:

$$R : \frac{[\ \neg p(X), p(s(X))\ ] \qquad\qquad [\ \neg p(Y), p(s(Y))\ ]}{[\ \neg p(X), p(s(s(X)))\ ]}$$

The following clause set S is a set of instances of $[\ \neg p(X), p(s(X))\ ]$.

$[\ \neg p(0),\ p(s(0))\ ]$,
$[\ \neg p(s(0)),\ p(s(s(0)))\ ]$,
$[\ \neg p(s(s(0))),\ p(s(s(s(0))))\ ]$,
$[\ \neg p(s(s(s(0)))),\ p(s(s(s(s(0)))))\ ]$,
$[\ \neg p(a),\ p(s(a))\ ]$,
$[\ \neg p(s(a)), p(s(s(a)))\ ]$,

Construct all clauses that are derivable by resolution from S in a single step. Give for each possible derivation the substitution Θ that makes it an instance of R. (Such substitution always exists, due to the lifting theorem)

3. Skolemize the following formulas. If required, transform them into Negation Normal Form first:

   (a) $\forall x \exists y\ x < y$.

(b) $\forall x\ A(x) \rightarrow (\ \exists y\ B(y) \wedge x = y)$

(c) $\forall x\ (\ (\ \exists y\ P(x,y)\ ) \rightarrow (\ \exists z\ P(x,z)\ )\ ).$

(d) $\forall x\ A(x) \rightarrow \exists y_1\ (\ R(x,y_1) \wedge \exists y_2\ (\ R(y_1,y_2) \wedge \exists y_3\ R(y_2,y_3)\ )\ ).$

(e) $\neg \exists xy\ (x < y \wedge y < x).$

4. Consider the sequent $\Gamma,\ \forall x\ (\neg p(x) \vee p(s(x))\ ) \vdash$

   By resolution, one can obtain the sequent $\Gamma,\ \forall x\ (\neg p(x) \vee p(s(x))\ ), \forall x\ (\neg p(x) \vee p(s(s(x)) \vdash$ from it.

   Show that if the second sequent has a proof, then the first one also has. Do this by proving the first sequent from the second. You will need cut.

5. Consider the sequent

$$\forall x\ \exists y\ P(x,y) \vdash \forall x\ \exists y_1 y_2\ (\ P(x,y_1) \wedge P(y_1,y_2)\ )$$

   (a) Make the sequent one sided, and transform it into NNF.
   (b) Apply Skolemization.
   (c) Transform into clausal normal form.
   (d) Use resolution to prove the resulting sequent.

6. Consider the sequent

$$\vdash\ (\ \exists x\ P(x)\ ) \leftrightarrow (\ \exists x\ P(x) \wedge Q(x)\ ) \vee (\ \exists x\ P(x) \wedge \neg Q(x)\ ).$$

   (a) Make the sequent one sided, and transform it into NNF.
   (b) Apply Skolemization.
   (c) Transform into clausal normal form.
   (d) Use resolution to prove the resulting sequent.