Theorem Proving (List 3)

Deadline: 16.03.2016

1. Use Knuth-Bendix completion (ground superposition) to show that

   (a) $a \approx b,\ b \approx c,\ c \approx d \models a \approx d$,

   (b) $x + y \approx y + x,\ x + s(y) \approx s(x + y),\ s(y) + x \approx s(y + x) \models x + s(y) \approx s(y) + x$,

   (c) $a \approx s(s(a)),\ s(s(a)) \approx s(t(a)),\ t(a) \approx b,\ t(s(b)) \approx c \models s(c) \approx a$.

2. Let $\succ$ be the following order: (The real Knuth-Bendix order)

   (a) If $\#t_1 > \#t_2$, then $t_1 \succ t_2$.

   (b) If $\#t_1 = \#t_2$, then write t_1 in the form $t_1 = f(\alpha_1, \ldots, \alpha_n)$, and write t_2 in the form $t_2 = g(\beta_1, \ldots, \beta_m)$.

      i. If $f \succ g$, then $t_1 \succ t_2$.

      ii. If $f = g$, then let i be the smallest index for which $\alpha_i \not\approx \beta_i$. If $\alpha_i \succ \beta_i$, then $t_1 \succ t_2$.

   Show that $\succ$ from the previous question is a reduction order.

3. Using KBO based on $a \succ b \succ c \succ f$, determine in each clause the maximal equation, and the directions in which the equations will be applied.

   (a) $[\ a \approx b,\ a \approx c\ ]$

   (b) $[\ a \approx b,\ f(a) \approx f(b)\ ]$.

   (c) $[\ a \approx b,\ a \not\approx c\ ]$

   (d) $[\ a \not\approx b, a \not\approx b,\ a \approx c\ ]$

   (e) $[\ a \not\approx b,\ a \approx c\ ]$.

4. Using KBO with $a \succ b \succ c$, find a refutation for the following clause set:

   $$[\ a \approx b,\ a \approx c\ ]$$
   $$[\ b \approx c\ ]$$
   $$[\ a \not\approx b,\ a \not\approx c\ ]$$

   Can you find a refutation without equality factoring?

5. Consider the following (satisfiable) clause set:

$$[\ A, B, C\ ]$$
$$[\ \neg A, B, C\ ]$$
$$[\ \neg B\ ]$$

Translate it into equational logic. Saturate the clause set, using $A \succ B \succ C$.