$G^{k,l}$-constrained multi-degree reduction of Bézier curves

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Abstract

We present a new approach to the problem of $G^{k,l}$-constrained ($k, l \leq 3$) multi-degree reduction of Bézier curves with respect to the least squares norm. First, to minimize the least squares error, we consider two methods of determining the values of geometric continuity parameters. One of them is based on quadratic and nonlinear programming, while the other uses some simplifying assumptions and solves a system of linear equations. Next, for prescribed values of these parameters, we obtain control points of the multi-degree reduced curve, using the properties of constrained dual Bernstein basis polynomials. Assuming that the input and output curves are of degree n and m, respectively, we determine these points with the complexity $O(mn)$, which is significantly less than the cost of other known methods. Finally, we give several examples to demonstrate the effectiveness of our algorithms.

Keywords: Constrained dual Bernstein basis, Bézier curves, Multi-degree reduction, Geometric continuity, Quadratic programming, Nonlinear programming.

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