Construction of dual bases

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Abstract

Let $B_n := \{b_0, b_1, \ldots, b_n\}$ $(n = 0, 1, \ldots, N;\ N \in \mathbb{N})$ be the sets of linearly independent functions. We give a simple method of construction the dual functions $D_n := \left\{d_0^{(n)}, d_1^{(n)}, \ldots, d_n^{(n)}\right\}$ $(0 \leq n \leq N)$ satisfying the following conditions: $\operatorname{span} D_n = \operatorname{span} B_n$ and $\left\langle b_i, d_j^{(n)} \right\rangle = \delta_{ij}$ $(0 \leq i, j \leq n \leq N)$, where $\delta_{ii} = 1$, $\delta_{ij} = 0$ for $i \neq j$, and $\langle \cdot, \cdot \rangle$ is a given inner product. The proposed algorithm allows us to construct all the sets of the dual functions $D_0, D_1, \ldots, D_N$ in the time $O(N^3)$, where N is a natural number. Four illustrative examples presenting the possible applications of obtained results are given.

Keywords: Dual bases; Approximation; Bernstein basis polynomials; B-spline functions.

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[1]Supported by Narodowe Centrum Nauki (Poland) under the grant 2011/01/B/ST1/01221