OPTYMALNA NUMERYCZNA APROKSYMACJA KLAS KAWAŁKAMI HÖLDEROWSKICH

OPTIMAL NUMERICAL APPROXIMATION OF PIECEWISE HÖLDER CLASSES

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Abstract. We study the L^p-approximation of scalar functions f consisting of two smooth pieces separated by an unknown singular point s_f; each piece is r times differentiable and rth derivative is Hölder continuous with exponent ρ. Allowed approximations use n inexact function values $y_i = f(x_i) + e_i$ with $|e_i| \le \delta$. Let $1 \le p < \infty$. We show that then the minimal worst case error is proportional to $\max(\delta, n^{-(r+\rho)})$ in the class of functions with uniformly bounded both the Hölder coefficients and the discontinuity jumps $|f(s_f^+) - f(s_f^-)|$. This error is achieved by an algorithm that uses a new adaptive mechanism to approximate s_f, where the number of adaptively chosen points x_i is only $\mathcal{O}(\ln n)$. The use of adaption, $p < \infty$, and the uniform bound on the Hölder coefficients and the discontinuity jumps are crucial. If we restrict the class even further to globally continuous functions, then the same worst case result can be achieved also for $p = \infty$ using no more than $(r-1)_+$ adaptive points.

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