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Multivariate generalized Bernstein polynomials. Identities for orthogonal polynomials of two variables

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Abstract. We introduce multivariable generalized Bernstein polynomials

$$B_{\boldsymbol{k}}^{n}(\boldsymbol{x};\omega|q) := \frac{1}{(\omega;\,q)_n} \begin{bmatrix} n \\ \boldsymbol{k} \end{bmatrix}_q \prod_{m=1}^{d+1} x_m^{k_m} \left(\frac{x_{m-1}}{x_m};\,q\right)_{k_m} \qquad (0 \le |\boldsymbol{k}| \le n),$$

of total degree $n \in \mathbb{N}$, where $\boldsymbol{k} = (k_1, \ldots, k_d) \in \mathbb{N}_0^d$, $0 \le k_1 + \ldots + k_d \le n$, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_d) \in \mathbb{R}^d$, depending on two parameters q and ω, which generalize the multivariate classical and discrete Bernstein polynomials. (We use the standard notation for the q-binomial coefficient, and the q-Pochhammer symbol). For $\omega = 0$, we obtain an extension of univariate q-Bernstein polynomials, introduced by Phillips (Ann. Numer. Math. **4** (1997)). Basic properties of the new polynomials are given, including recurrence relations, q-differentiation rules and de Casteljau algorithm. For the case $d = 2$, connections between $B_{\boldsymbol{k}}^{n}(\boldsymbol{x};\omega|q)$ and bivariate orthogonal big q-Jacobi polynomials—introduced recently by the first two authors—are given, with the connection coefficients being expressed in terms of bivariate q-Hahn polynomials. As limiting forms of these relations, we give connections between bivariate q-Bernstein and Dunkl's (little) q-Jacobi polynomials (SIAM J. Alg. Disc. Meth. **1** (1980)), as well as between bivariate discrete Bernstein and Hahn polynomials.

Keywords: Multivariate generalized Bernstein polynomials, Multivariate q-Bernstein polynomials, Bivariate big q-Jacobi polynomials, Bivariate q-Hahn polynomials, Bivariate Hahn polynomials, Connection relations.

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Dedicated to the memory of Luigi Gatteschi

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