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Dual generalized Bernstein basis

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Abstract. The generalized Bernstein basis in the space Π_n of polynomials of degree at most n, being an extension of the q-Bernstein basis introduced recently by G.M. Phillips, is given by the formula (see S. Lewanowicz & P. Woźny, BIT 44 (2004), 63–78)

$$B_i^n(x;\omega|\,q) := \frac{1}{(\omega;\,q)_n}\begin{bmatrix} n \\ i \end{bmatrix}_q x^i\,(\omega x^{-1};\,q)_i\,(x;\,q)_{n-i} \qquad (i=0,1,\ldots,n).$$

We give explicitly the dual basis functions $D_k^n(x;a,b,\omega|\,q)$ for the polynomials $B_i^n(x;\omega|\,q)$, in terms of big q-Jacobi polynomials $P_k(x;a,b,\omega/q;q)$, a and b being parameters; the connection coefficients are evaluations of the q-Hahn polynomials. An inverse formula – relating big q-Jacobi, dual generalized Bernstein, and dual q-Hahn polynomials – is also given. Further, an alternative formula is given, representing the dual polynomial D_j^n $(0 \le j \le n)$ as a linear combination of $\min(j,n-j)+1$ big q-Jacobi polynomials with shifted parameters and argument. Finally, we give a recurrence relation satisfied by D_k^n, as well as an identity which may be seen as an analogue of the extended Marsden's identity.